REAL OPTIONS IN PARTNERSHIP DEALS: THE PERSPECTIVE OF COOPERATIVE GAME THEORY

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Abstract. In this paper we consider partnership deals under uncertainty but with downstream flexibility. We confine ourselves to bilateral deals and focus on the effect of options on the synergy set, the 'core', of a partnership deal. We distinguish between cooperative options, which are exercised jointly and in the interest of maximizing the total deal value, and non-cooperative options, which are exercised unilaterally in the interest of one partner's payoff. We provide a simple framework that illustrates options effects in a two-stage model. The model can be readily extended to a binomial lattice. We investigate options effects in the presence of risk-aversion and in the presence of complete markets.

1. Introduction

Partnership deals are a driving force of the modern economy. Joint ventures of car manufacturers in developing economies, alliances between airlines, co-development deals between pharmaceutical and biotech companies, production sharing contracts between oil majors and national oil companies, to name but a few industries where partnership deals are significant drivers for value. Partnerships aim to establish synergies by combining core competencies of the partners to form a unique offering that neither partner could provide alone.

Structuring successful partnership deals is an immensely difficult task and arguably more an art than a science. Nevertheless, science can help to shed light on deal values. Two key questions are: How should the contract be structured to generate significant *total value* at an acceptable level of risk? How should this total value and the associated

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risks be *shared* amongst the partners?

The academic discussion on the distribution of benefits from cooperation has been inspired by the seminal work of Nash [12, 13] and Shapley [16], which led to the advent of bargaining theory and cooperative game theory. This literature is largely concerned with the allocation of value, not of risk. The models are mainly deterministic and combinatorial. A first strand of this literature relevant to our work is concerned with cooperative game theory in the presence of stochastic payoffs, see e.g. Granot [5] and Suijs and Borm [19]. A second relevant strand of the literature focuses on efficient risk sharing and the formation of syndicates, see e.g. Wilson [21] and Pratt [14]. We integrate these two strands of literature with elements of the real options literature in our study of the effect of optionality in partnership deals, a topic which, to our best knowledge, has not been thoroughly investigated before.

Generally speaking, explorations of effects of uncertainty are dominated by two mental concepts: Diversification and Optionality. Diversification is a *passive* risk management tool and presumes no direct influence on the management of projects or companies. It is therefore particularly appealing to investors and is the key concept behind the seminal work of Harry Markowitz [9] on portfolio choice and of William Sharpe [17] on the capital asset pricing model, for which they received the 1990 Nobel prize in economics.

In contrast, optionality is an *active* risk and opportunity management technique and is therefore particularly appealing to managers. Optionality is based on the idea that creating the right but not the obligation to a potential future action creates value in an uncertain environment. The concept of optionality and the valuation thereof in the context of financial derivatives has been the basis for the 1997 Nobel award to Robert Merton and Myron Scholes [10, 1], whose work with the late Fisher Black has created the field of financial engineering. Indeed, their ideas have moved beyond the realm of financial derivatives into capital budgeting and project valuation. Steward Myers [11] was amongst the first to advocate that significant optionality, such as growth opportunities, ought to be included in the valuation of a project or company and that appropriate use of the work of Black, Scholes and Merton might make this possible. Myers saw this as an opportunity to bridge the gap between strategy and finance and coined the term 'real options' for this line of thinking. Shortly afterwards, Brennan and Schwarz [2] illustrated how such real options could be valued with a Black-Scholes approach. These seminal papers, together with the monograph by Dixit and Pindyck [4], who substantially elaborated the ideas, spurred a significant amount of academic work over the past two decades and led to the establishment of *real options* as a distinct area in finance with significant uptake in the strategy literature.

It is surprising that little work has been done to date to understand the effect of real options in partnership deals, in particular with regard to fair splits of risk and return from a partnership. Our aim in this paper is to fill this gap to some extent and to present a framework that allows the investigation of contract issues with a real options flavour. We approach this agenda by combining concepts from cooperative game theory and real options theory. Our main emphasis is on the impact of options on the core of a cooperative game. The core of a deal conceptualizes a notion of negotiation set or synergy set. It comprises of those allocations of the total deal value to the players for which no player can do better by not agreeing to the deal. For the sake of simplicity we will focus on bilateral partnerships.

2. Options deals: Cooperative vs. non-cooperative options

An *options deal* is a partnership deal with significant future flexibility. Options are rights but not obligations to future actions. In a partnership situation this raises the question, who has the right to the action? There are essentially two types of options deals, depending on who has this exercise right:

(1) In pure partnership deals exercise decisions are taken jointly and in the interest of maximizing the total value of the deal. We call such flexibilities 'cooperative options'. A typical example is a decision to jointly market a product after a successful R&D effort.
(2) In contrast, some flexibility might be privately owned by one of the partners, who has the right to exercise it in the interest of his or her own payoff, rather than the sum of payoffs resulting from the deal. We call such flexibilities 'non-cooperative options'. A generic non-cooperative option on a deal is to renege if circumstances do not unfold as anticipated, accepting possible litigation costs as the price of exercise.

The notion of cooperative options emphasizes the collaborative nature of partnerships, whilst non-cooperative options acknowledge the transient nature of deals and regard them as part of competitive strategies of firms who will ultimately act in self-interest. Non-cooperative options can be explicitly acknowledged in a partnership contract. For example, a clause in a co-development contract between a biotech and a pharmaceutical company may allow the biotech company to opt out of further co-development and receive agreed milestone and royalty payments instead. The smaller company may want to exercise this option if the costs of further development become prohibitively large or promising candidate drugs have emerged further upstream in the company's R&D pipeline and are regarded a better use of the company's capital. Furthermore, as we shall see later, the presence of non-cooperative options reduce the synergy set in favour of the option holder and thereby steer the outcome of a negotiation away from less desirable sharing arrangements.

Non-cooperative options in deals can be thought of as a cooperative game followed by a non-cooperative game. The cooperative game, i.e., the deal contract, sets the framework for later non-cooperative behaviour, which has to be taken into account in the design and valuation of the deal.

Before we investigate issues around non-cooperative options in more detail, we will discuss cooperative options, which by their very nature fall into the remit of cooperative game theory. Cooperative options are exercised to maximize the total deal value. Therefore, deal valuation and optimal exercise issues are similar to standard real options analysis. However, cooperative options can have an impact on the negotiation set, also known as the *core* or the synergy set of the deal, which is the set of allocations of the payoffs to the partners that will make *all* partners better off than without the deal. We illustrate this fact by way of a stylized example. This example will also serve as a gentle introduction to real options arguments for a cooperative game theory audience and to concepts from cooperative game theory for readers from the real options community.

2.1. **The core of an options deal: An illustrative example.**

2.1.1. *Setting.* Assume a biotech company has a drug under development, which has successfully passed the clinical trials and is now awaiting FDA approval. The company estimates the present value of cash flows from the drug to be C_B for a launch investment of $I_B < C_B$. The biotech company has limited production capabilities and its sales and distribution network is rather inefficient compared to the major players in the market. The company is therefore negotiating a co-marketing deal with a large pharmaceutical company. The cash flow projection for the co-marketed product is C_{B+P} and the launch investment will be I_{B+P}. How should the value $(C_{B+P} - I_{B+P})$ of the deal be shared in a fair way?

2.1.2. *The core of the game.* The core of this cooperative game is the set of revenue allocations that make both partners better off than going alone. By going alone, the biotech company makes an estimated profit of $C_B - I_B$, while the pharmaceutical company receives nothing. If x_B and x_P denote the share of the deal value for the biotech and pharmaceutical company, respectively, and neglecting opportunity costs of capital considerations for simplicity, the conditions for the core are

$$\begin{aligned} x_B &\geq C_B - I_B \\ x_P &\geq 0 \\ x_B + x_P &= C_{B+P} - I_{B+P}. \end{aligned}$$

In other words, a biotech profit share x_B is in the core if

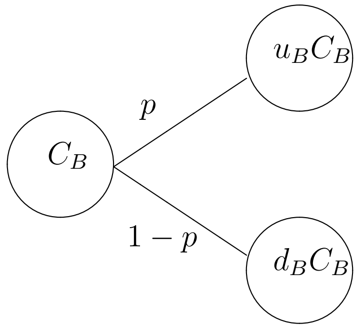
$$C_B - I_B \leq x_B \leq C_{B+P} - I_{B+P},$$

with the residual profit $x_P = C_{B+P} - I_{B+P} - x_B$ being allocated to the pharmaceutical company. The precise sharing arrangement will be a matter of negotiation, but whatever the negotiation result, it should lie in the core.

2.1.3. *The core of a game with uncertain payoffs.* Assume a competitor is developing a drug that will treat the same indication. If the competitor is successful in bringing its drug to the market, the revenue potential of the biotech's drug will be greatly reduced. In the upside scenario of failure of the competitor, the cash flow projection for the drug if the biotech company goes alone rises to $u_B C_B$, with $u_B > 1$; in the downside scenario of a competitor success, this projection is only $d_B C_B$, with $d_B < 1$. We assume that p is the chance of failure of the competing drug and, for simplicity, that the biotech company has zero risk aversion, i.e. it is indifferent to an amount of money for sure or a gamble with the same expected payoff. To make sure that the initial valuation of C_B for the future cash flows is consistent with the scenario assumptions, we require that

$$u_B = 1 + s_B\sqrt{\frac{1-p}{p}},\ d_B = 1 - s_B\sqrt{\frac{p}{1-p}}, \tag{1}$$

where s_B is a measure of volatility[1]. The cash flow uncertainty is easily depicted in a scenario fork:



In the partnership deal the present values of the cash flows are projected as $u_{B+P}C_{B+P}$ in the upside and $d_{B+P}C_{B+P}$ in the downside scenarios. Again, under the zero risk aversion assumption, this scenario assumption is consistent with the foregoing valuation of C_{B+P} if the upwards and downwards multipliers u_{B+P}, d_{B+P} have the form (1) with a possibly different volatility s_{B+P}.

An allocation in a game with uncertain payoffs must specify the payoff allocations for every future scenario. Let us denote by $x_{B,u}, x_{B,d}$ and $x_{P,u}, x_{P,d}$ the allocations for the biotech and pharmaceutical company in the upside and downside scenarios, respectively. In view of the zero risk aversion assumption, a profit allocation is in the core if the following conditions hold

$$\begin{aligned} px_{B,u} + (1-p)x_{B,d} &\geq C_B - I_B \\ px_{P,u} + (1-p)x_{P,d} &\geq 0 \\ x_{B,u} + x_{P,u} &= u_{B+P}C_{B+P} - I_{B+P} \\ x_{B,d} + x_{P,d} &= d_{B+P}C_{B+P} - I_{B+P}. \end{aligned}$$

The first two conditions guarantee that each player's valuation of his or her share of the profit is at least as large as the value from going alone. The final two conditions guarantee that the sharing agreements

[1] A geometric Brownian motion with drift ν and volatility σ is approximated by a binomial lattice with upwards probability p, period length Δt and upwards and downwards multipliers $u = \exp\left(\nu\Delta t + \sigma\sqrt{\Delta t}\sqrt{\frac{1-p}{p}}\right)$ and $d = \exp\left(\nu\Delta t - \sigma\sqrt{\Delta t}\sqrt{\frac{p}{1-p}}\right)$, respectively, see Luenberger [8], p. 314. The simplified form (1) is a first order approximation of the latter formulas for small $s_B = \sigma\sqrt{\Delta t}$ and $\nu = 0$.

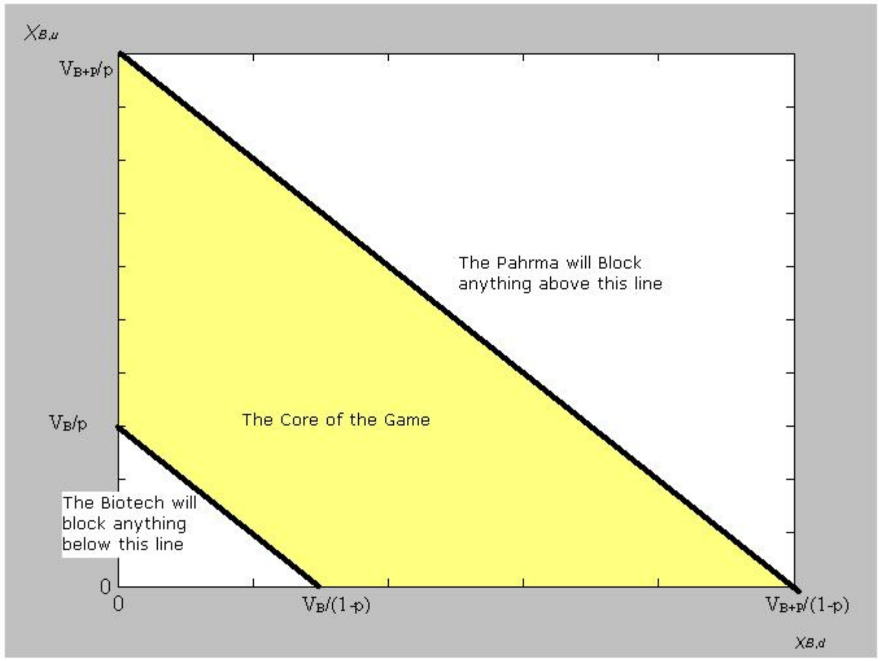


FIGURE 1. The core of the Cooperative Game

sum up to the total value in each scenario.

Eliminating $x_{P,u}$ and $x_{P,d}$ through the two equations results in a representation of the core in $(x_{B,u}, x_{B,d})$-space:

$$C_B - I_B \leq p x_{B,u} + (1-p) x_{B,d} \leq C_{B+P} - I_{B+P}. \tag{2}$$

The final equation is a consequence of the assumption that u_{B+P}, d_{B+P} satisfy a relation of the form (1), with B replaced by $B+P$.

The core is a strip in $(x_{B,u}, x_{B,d})$-space, as depicted in Figure 1. Note that under the zero risk aversion assumption the biotech company is indifferent between revenue allocations on a line $p x_{B,u} + (1-p) x_{B,d} = V$ and values them all at the expected value V. The same is true for the pharmaceutical company, which assigns to these sharing arrangements the residual value $C_{B+R} - V$. We will see later in the paper that more interesting risk-return tradeoffs occur in the presence of risk aversion.

2.2. **A cooperative option.** Next suppose that the companies can wait with the launch investment until they know the result of the trials for the competing drug and therefore the cash flow scenario. For simplicity we assume that there is no cost of waiting. In this case, the companies can avoid a loss in the downside case by not launching the drug. Under risk-neutrality, the value of the drug for the biotech company alone is now

$$V_B = p\max\{u_B C_B - I_B, 0\} + (1-p)\max\{d_B C_B - I_B, 0\}, \tag{3}$$

while the value for the partnership becomes

$$V_{B+P} = p\max\{u_{B+P} C_{B+P} - I_{B+P}, 0\} + (1-p)\max\{d_{B+P} C_{B+P} - I_{B+P}, 0\}. \tag{4}$$

The conditions for the core are

$$\begin{aligned} px_{B,u} + (1-p)x_{B,d} &\geq V_B \\ px_{P,u} + (1-p)x_{P,d} &\geq 0 \\ x_{B,u} + x_{P,u} &= \max\{u_{B+P}C_{B+P} - I_{B+P}, 0\} \\ x_{B,d} + x_{P,d} &= \max\{d_{B+P}C_{B+P} - I_{B+P}, 0\}. \end{aligned}$$

In $(x_{B,u}, x_{B,d})$-space the core moves to

$$V_B \leq px_{B,u} + (1-p)x_{B,d} \leq V_{B+P}.$$

In other words the geometrical shape of the core has not changed but the option has shifted the core and may have changed its diameter.

To illustrate this effect, let us assume that

$$\begin{aligned} \max\{d_B C_B, d_{B+P}C_{B+P}\} &\leq \min\{I_B, I_{B+P}\} \\ \min\{u_B C_B, u_{B+P}C_{B+P}\} &\geq \max\{I_B, I_{B+P}\}, \end{aligned}$$

i.e., neither the biotech alone, nor the partnership will launch in the downside scenario but both will launch in the upside scenario. In this case the total values V_B and V_{B+P} of the go-alone project and the deal are

$$\begin{aligned} V_B &= p(u_B C_B - I_B) \\ V_{B+P} &= p(u_{B+P}C_{B+P} - I_{B+P}). \end{aligned} \tag{5}$$

Following Trigeorgis [20, 18], we find it convenient to express the total value as the sum of an 'asset value' (or 'passive value') and an 'option value'. The total asset values, i.e. the values without flexibility, were calculated above as $V_B^A = C_B - I_B$ and $V_{B+P}^A = C_{B+P} - I_{B+P}$, respectively, which leaves as total option values

$$\begin{aligned} V_B^O &= (pu_B - 1)C_B + (1-p)I_B \\ V_{B+P}^O &= (pu_{B+P} - 1)C_{B+P} + (1-p)I_{B+P}. \end{aligned}$$

The core of this game with a cooperative option is

$$V_B \le p x_{B,u} + (1-p) x_{B,d} \le V_{B+P}. \tag{6}$$

It is possible to represent this core as the sum of two cores, an 'asset core', which does not take optionality into account, and an 'options core'. Using (1),(5) and dividing by p we obtain the equivalent core representation

$$C_B - I_B + s_B\sqrt{\frac{1-p}{p}}C_B \le x_{B,u} + \frac{1-p}{p}x_{B,d} \le C_{B+P} - I_{B+P} + s_{B+P}\sqrt{\frac{1-p}{p}}C_{B+P}.$$

We split the biotech's allocation into two components $x_{B,\omega} = x^A_{B,\omega} + x^O_{B,\omega}$, where $\omega \in \{u, d\}$. The first component $x^A_{B,\omega}$ is the biotech's share of the total 'asset value' (or 'passive value') of the deal, the second component allocation $x^O_{B,\omega}$ is its share of the total 'option value' of the deal. The core for the asset component is the core of the game without the option, i.e.,

$$C_B - I_B \le p x^A_{B,u} + (1-p)x^A_{B,d} \le C_{B+P} - I_{B+P}.$$

In view of the above representation of the core for the aggregate allocation $(x_{B,u}, x_{B,d})$ the core of the option component $x^O_{B,\omega} = x_{B,\omega} - x^A_{B,\omega}$ becomes

$$s_B\sqrt{\frac{1-p}{p}}C_B \le (1-p)x^O_{B,u} + \frac{(1-p)^2}{p}x^O_{B,d} \le s_{B+P}\sqrt{\frac{1-p}{p}}C_{B+P},$$

which is equivalent to

$$s_B\sqrt{\frac{p}{1-p}}C_B \le p x^O_{B,u} + (1-p)x^O_{B,d} \le s_{B+P}\sqrt{\frac{p}{1-p}}C_{B+P}.$$

The options core is independent of the investments I_B or I_{B+P} and is empty if

$$s_B C_B \ > \ s_{B+P} C_{B+P}.$$

2.2.1. *The effect of volatility on the core of a cooperative option.* Recall that the lower bound is the minimal share of the total option value of the deal that the biotech might accept, whilst the upper bound is the maximal share that it can expect. In other words, the larger the lower bound, the better the negotiation position for the biotech company, the larger the upper bound, the better the negotiation position for both companies.

The upper bound of the biotech's core increases with increasing deal volatility s_{B+P}, giving both players more synergies to negotiate over. The lower bound of the biotech's options core grows, ceteris paribus, linearly with the volatility s_B of the go-alone revenue estimate. The larger that volatility, the more the core shrinks. It shrinks by pushing out lower values for the biotech, i.e. an increase of the biotech volatility is favourable for the biotech company. This is because the increased go-alone volatility increases the biotech's go-alone value but not the value of the deal, assuming the deal volatility remains constant.

2.2.2. *Linear contracts.* A specific type of contract that is often encountered in practice pays a fixed amount β and a share α of the revenues. The corresponding allocations of the total deal value $C_{B+P} - I_{B+P}$ for the biotech company are

$$\begin{aligned} x_{B,u} &= \alpha u_{B+P} C_{B+P} + \beta \\ x_{B,d} &= \alpha d_{B+P} C_{B+P} + \beta. \end{aligned}$$

The core (6) of the options deal in terms of the new parameters α, β becomes

$$V_B \leq \alpha C_{B+P} + \beta \leq V_{B+P}.$$

This illustrates the tradeoff of shares in the risky revenues against receiving a fixed amount. If we fix the royalty rate α, we are left with negotiations over the share β of the investment costs within the core

$$V_B - \alpha C_{B+P} \leq \beta \leq V_{B+P} - \alpha C_{B+P}.$$

We will return to linear contracts later in the paper, when we deal with the effect of risk aversion.

2.3. **A non-cooperative option.** Let us now assume that the deal gives the biotech company unilateral flexibility to opt out of co-marketing of the drug before committing to the launch cost, whilst the pharma company is locked into the deal. Assume the biotech would receive a fixed amount Z if it exercised the opt-out option. The option would be exercised only if the payoff it offered to the biotech is higher than the biotech's valuation of the agreed profit share. Assume the launch decision will be taken after the fate of the competitor drug is observed. In this case, the biotech payoff will be $\max\{x_{B,\omega}, Z\}$, where $\omega \in \{u, d\}$ is the observed scenario. Given agreed profit shares $x_{B,u}, x_{B,d}$ for the biotech company, the expected revenue for the biotech company is $p \max\{x_{B,u}, Z\} + (1-p) \max\{x_{B,d}, Z\}$; the pharmaceutical company receives the residual revenue. The conditions for the core in $(x_{B,u}, x_{B,d})$-space are now

$$\begin{aligned} p\max\{x_{B,u}, Z\} + (1-p)\max\{x_{B,d}, Z\} &\geq C_B - I_B \\ p(u_{B+P}C_{B+P} - I_{B+P} - \max\{x_{B,u}, Z\}) & \\ +(1-p)(d_{B+P}C_{B+P} - I_{B+P} - \max\{x_{B,d}, Z\}) &\geq 0; \end{aligned}$$

the first condition ensures that the biotech has an incentive to participate in the deal, the second condition guarantees the same for the pharmaceutical company. The core in $(x_{B,u}, x_{B,d})$ space is therefore

$$C_B - I_B \leq p\max\{x_{B,u}, Z\} + (1-p)\max\{x_{B,d}, Z\} \leq C_{B+P} - I_{B+P}.$$

Assuming that $x_{B,d} \leq x_{B,u}$, i.e. the biotech gets at least as much in the upside scenario as in the downside scenario, there are three cases, depending on the value of Z and the agreed sharing arrangement $(x_{B,d}, x_{B,u})$:

(1) If $Z \leq x_{B,d}$ then the biotech will not exercise the option in any of the two scenarios. The core of the deal coincides with the core if there is no option:

$$C_B - I_B \leq px_{B,u} + (1-p)x_{B,d} \leq C_{B+P} - I_{B+P}.$$

(2) If $x_{B_d} < Z \leq x_{B,u}$ then the biotech will only exercise in the downside scenario and the core becomes

$$C_B - I_B \leq px_{B,u} + (1-p)Z \leq C_{B+P} - I_{B+P}.$$

(3) If $Z > x_{B,u}$ then the opt-out option will be exercised in both scenarios. The core condition then reduces to a condition for feasible Z values:

$$C_B - I_B \leq Z \leq C_{B+P} - I_{B+P}.$$

Figure 2 illustrates the options effect on the core of the game. The core is now the shaded area. Three distinct cases can arise:

(1) The Opt-out payoff lines intercept below (South West of) the core. In such a case the option does not really change anything for the risk neutral agents. It does however shrink the negotiation set in the sense that the payoffs in each state can no longer be too different from each other.
(2) The Opt-out lines intercept in the core. This is depicted in the figure 2. For this to happen the lump sum is higher than the payoff of the lower state but less than the payoff of the higher state. The change in the core is in the favour of the option owner.

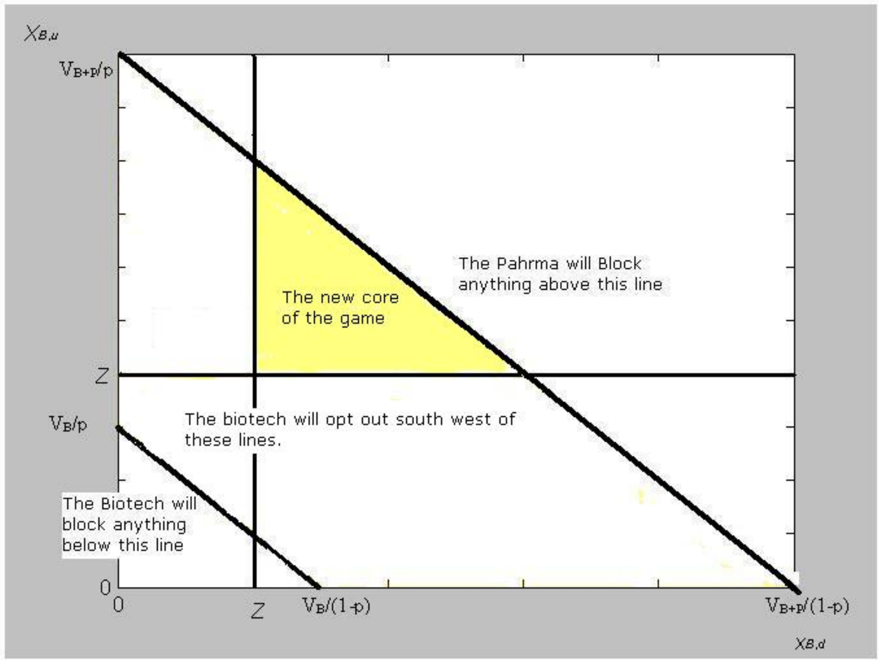


FIGURE 2. The core of the cooperative game with the Biotech having the Opt-out option

(3) The Opt-out lines intercept above (North East of) the core. In this case the core is empty. This happens only if the opt-out payoff is higher than the payoff of the up state payoff.

2.3.1. *A non-cooperative option for the pharma company.* An example of downstream flexibility for the pharma company would be the right for the pharma to buy out the biotech for a fixed sum S. This option would be exercised only if the the payoff from exercising is higher than the agreed payoff $x_{P,\omega}$ making the total payoff to the Pharma equal $C_{B+P} - I_{B+P} - \min\{X_{B,\omega}, S\}$. This option would reshape the core as shown in figure 3.
Again we can identify three distinct cases:

(1) The Buy-out option doesn't restrict the core for the risk neutral players but brings the payoffs of the two states closer together.
(2) The Buy-out lines restrict the core. This happens if the Buy-out lines intercept higher than the minimum line the biotech is willing to accept. This is depicted in figure 3.
(3) The core is empty. This happens if the Buy-out lines intercept lower than the minimum line the biotech is willing to accept.

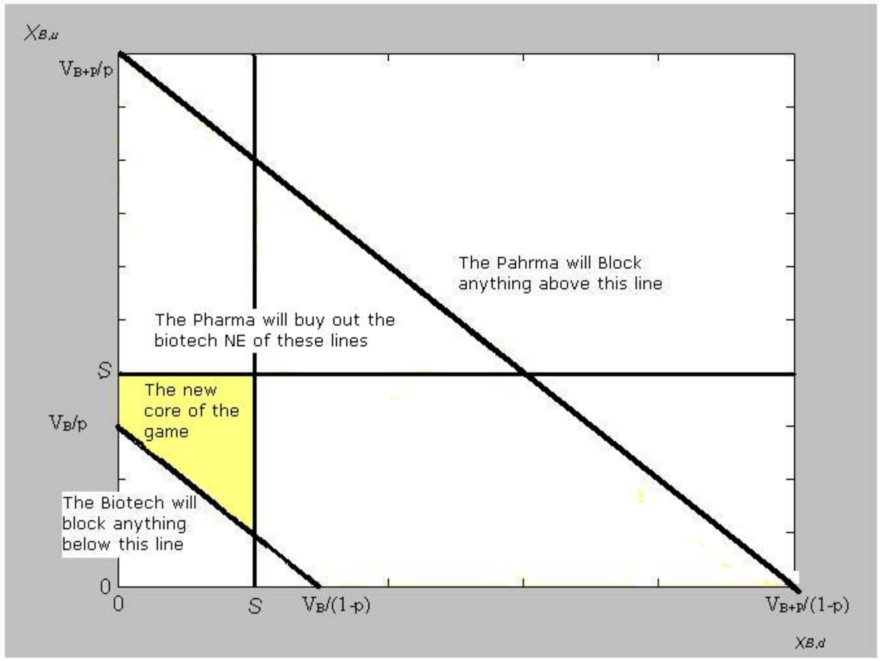


FIGURE 3. The core of the cooperative game with the Pharna having the Buy-out option

2.3.2. *Compound non-cooperative options.* Some more interesting situations might arise if both players have options which they exercise non-cooperatively. For example, the biotech may have the right to decide whether to opt out or not. If it decides not to opt out the pharma will decide if it wants to buy them out or not. Such sequential options can be dealt with in a similar way as above, evaluating the phases backwards in time for all scenarios:

Phase 4: Each agent exercises her options egoistically

Phase 3: The partnership exercises options jointly that maximise the total value of the project

Phase 2: Uncertainty is resolved

Phase 1: The agents form a coalition, decide on the optimal course of action (including a contingency plan) and decide how to split the payoffs.

The described example is a situation where options are exercised sequentially. In other situations, agents may have to exercise downstream flexibility simultaneously. This would give rise to a non cooperative game on the core of the cooperative game, which would go beyond the scope of this paper.

3. The effect of risk aversion

The presence of possibly different levels of risk aversion introduces two interesting issues:

(1) If two players have different levels of risk aversion they should be willing to tradeoff risk. This brings up the question how can risk be shared in an efficient way?
(2) If two players have different levels of risk aversion, they may well come to different conclusions about optimal options exercise of a cooperative option. How can these differing preferences be reconciled?

We address these issues in this section.

We will focus on linear contracts, i.e., agreements involving a share of a certain payment, e.g. a share of known investment costs, and a possibly different share of an uncertain payoff, e.g. royalties on uncertain revenues. Sensible sharing arrangements of the risky payoff should depend on the level of risk aversion of the players. It is natural to expect the less risk averse agent to take on more risk. Agents who take on more risk will rightly ask for compensation, in terms of the certain payment, because they are providing a type of insurance for the more risk averse partner. This can result in a win-win situation, i.e., the synergies that drive a partnership may well stem from differing levels of risk aversion. The deterministic payment depends on the known value that each player brings to the partnership, such as a reduction in investment cost through cooperation. Thus, the consideration of fixed deterministic payoff and shares in a gamble essentially decouples the 'stochastic' synergies obtained by exploiting differing risk aversions and the 'deterministic' synergies obtained through cooperation. It is known that a suitable linear risk sharing rule is a Pareto-efficient risk sharing agreement under certain assumptions on the agents' utility functions, see Pratt [14] or Christensen and Feltham [3].

We will illustrate the risk sharing issue for a simple two player partnership deal. We will assume that the stochastic payoff X of the deal follows a binomial process as in the previous examples. The two players have different attitudes towards risk: the first player is more risk averse than the second. We model their payoff preferences via expected utility functions: Player i prefers a possibly uncertain payoff X over an uncertain payoff Y if $\mathbf{E}[u_i(X)] > \mathbf{E}[u_i(Y)]$, where u_i is a suitable utility function. For illustrative purposes we choose negative exponential utility functions for the two players:

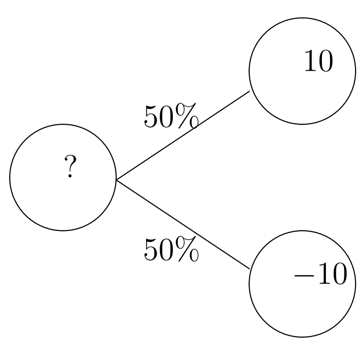
$$\mathbf{E}[u_1(X)] = -\mathbf{E}[e^{-\frac{X}{\beta_1}}]$$
$$\mathbf{E}[u_2(X)] = -\mathbf{E}[e^{-\frac{X}{\beta_2}}],$$

with $\beta_1 = 0.2 > \beta_2 = 0.1$. Since both players are risk averse, they would both prefer to receive the expected payoff of a risky gamble rather than take the gamble. Between two symmetric gambles with the same expectation, they would prefer the one with lower risk (lower variance).

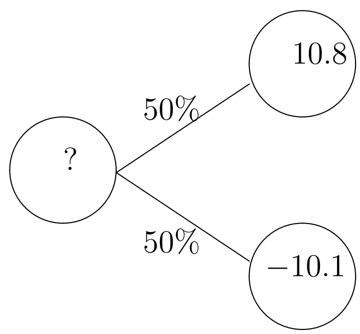
Since the players' perception of risk are fully captured by an expected utility, it is possible to gauge how much a risky gamble would be worth to each player. In other words, given a gamble X, what deterministic payment m_i would make agent i indifferent to receiving m_i for sure or taking the gamble? This value is called the *certainty equivalent* of the risky gamble. It satisfies $u_i(m_i) = \mathbf{E}[u_i(X)]$, i.e., $m_i = u_i^{-1}(\mathbf{E}[u_i(X)])$.

Suppose the two players have to choose between two projects, where project 1 has a lower risk but also a lower return, e.g.

Project1



Project2



Based on their utility functions, the first agent would prefer project 1 while the second agent would prefer project 2. Naturally, the certainty equivalent for player 1 from project 1 is higher than from project 2 and vice versa for player 2. So if the players had to select one project to develop in partnership they would disagree which one to choose.

What is the optimal way for the players to share the risk involved in such a project? As mentioned above, we will constrain ourselves to linear contracts, i.e., the agreement will involve a split that has a deterministic part d_i and a share r_i in the stochastic gamble. We assume that $d_1 + d_2 = 0$ and $r_1 + r_2 = 1$, $r_i \geq 0$. The total payoff of player i from the project will be $d_i + r_i X$, where X is the random payoff of the gamble. This is illustrated in figure 4; we are looking for an allocation along the diagonal in the space of state-payoffs. The closer we are to the origin, more risk is taken up by agent 2 and the less risk by agent 1. In the example of section 2 where players were risk neutral, their utility indifference curves were straight lines and utility for the biotech company was increasing in magnitude the further away these lines were from the origin. Furthermore, the utility curves were all parallel to each other, as can be seen in figure 1. Now the payers are risk averse and the indifference map is no longer linear as can be seen in figure 4.

Both players prefer to take up as little risk as possible. However, the risk aversion induced by the concave utility function reduces the marginal benefit of a decrease in risk taking. Furthermore, this rate of reduction of marginal benefits will be different for both players, in view of their differing risk aversion levels. However, in the context of a partnership, players cooperate to maximise the total value of the game. We model this objective by assuming that the players wish to

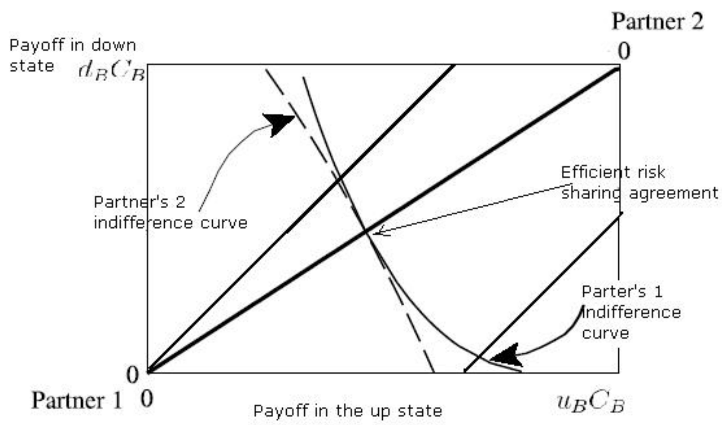


FIGURE 4. Efficient risk sharing

maximise the sum of their certainty equivalents. The optimality condition for the royalty rate r that maximises the perceived total value of the game is the point where the marginal value of taking up some infinitesimal fraction of the risky project is the same for both agents. If the marginal benefits were different, we would be able to add to the total value by taking away an infinitesimal amount of risk from the player with the smaller marginal benefit and giving it to the player with the larger marginal benefit. This can be seen in figure 5.

Formally, the agents solve the maximisation problem $\max_{r_1,r_2}(m_1(d+r_1X)+m_2(-d+(r_2)X)$. Since certainty equivalents satisfy $m(d+Y)=d+m(Y)$ for deterministic payoffs d and stochastic payoffs Y, the problem reduces to $\max_r(m_1(rX)+m_2((1-r)X)$. Here, we have also used the fact that $r_1+r_2=1$. The first order optimality condition is $\frac{dm_1(rX)}{dr}=-\frac{dm_2((1-r)X)}{dr}$. Although this condition specifies how much risk each player will take, it does not determine the total payoff, as the deterministic amount $d_1=-d_2$ that will change hands has not been decided yet. If the players agree to share risk in the optimal way, as defined above, then it is this amount d_1 that the players will have to negotiate over. The cooperative game is reduced to a deterministic

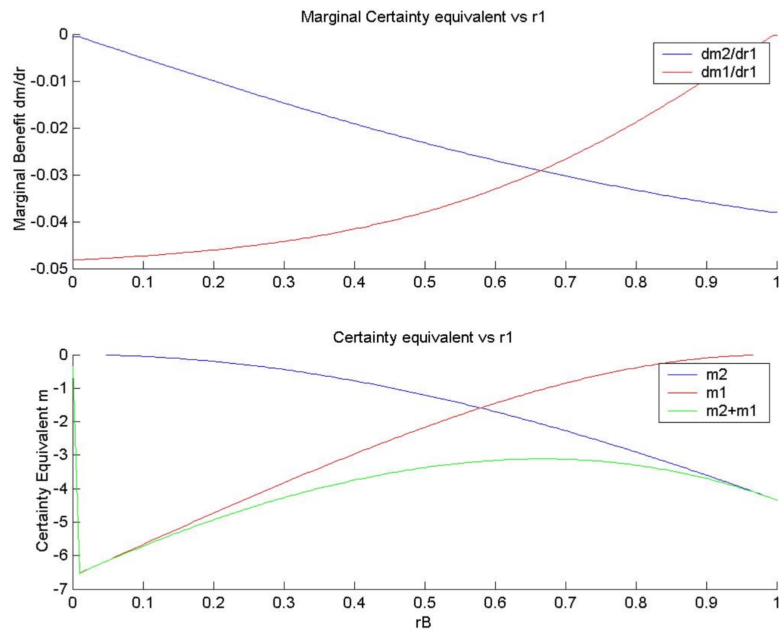


Figure 5. The risk sharing problem

game with a core in the standard sense.

Returning to the problem of selecting one of the two projects for a partnership, recall that agent 1 would prefer project 1 and agent 2 would prefer project 2. If the two agents agree to share the risk optimally, they would agree that the riskier project 2 is preferable. Note that the total value of both projects is higher when the agents are cooperating than when each agent goes alone. Cooperation is a win-win opportunity for them.

4. Cooperative options and risk aversion

4.1. **Optimal risk sharing.** As we demonstrate in the previous section, the presence of risk aversion does not complicate the situation dramatically, at least for so-called HARA utility functions, see [14]. Linear sharing rules allow an optimal sharing of the risk. Once the players agree that they wish to share risk optimaly, the cooperative game is played on the deterministic amount that the players will exchange.

Let us now go back to the example of Section 2.1. We will denote by r_B and r_P the shares of the risky project for the biotech and pharma company, respectively; m_B, m_P will be the respective certainty equivalents, and d_B, d_P will be the agreed certain payoffs with $d_P + d_B = 0$. We first need to find the efficient risk sharing agreement. In order to do this we have to solve the problem

$$\begin{array}{ll} \max\limits_{r_B, r_P} & m_B(r_B C_{P+B}) + m_P(r_P C_{P+B}) \\ \text{s.t.} & r_B + r_P = 1. \end{array}$$

This is a standard problem in the risk sharing literature [3, 21]. For the negative exponential utility function the optimal share of risk for each player is simply proportional to their risk tolerance:

$$r_i = \frac{\beta_i}{\sum_j \beta_j}.$$

4.2. **The core of the Cooperative game.** Following the example of section 2 the core of the game is now given by the following equations:

$$\begin{array}{rcl} m_B(r_B[C_{B+P} - I_{B+P}]) + d_B & \geq & m_B(C_B - I_B) \\ m_P(r_P[C_{B+P} - I_{B+P}]) + d_P & \geq & 0 \\ d_B + d_P & = & 0 \\ r_i & = & \dfrac{\beta_i}{\beta_B + \beta_P}, \;\; i \in \{B, P\}. \end{array}$$

The first two conditions guarantee that each agent's estimation of the value is at least as good as going alone. The third condition is a conservation law: the total amount that changes hands is zero and the forth condition will ensure efficient risk sharing.

The core is now one dimensional and involves only the deterministic amounts d_i that the two agents will exchange:

$$m_B(C_B - I_B) - m_B(r_B[C_{B+P} - I_{B+P}]) \leq d_B \leq m_P(r_P[C_{B+P} - I_{B+P}]). \tag{7}$$

The core is illustrated in Figure 6.

4.2.1. *The risk-sharing value of the partnership deal.* It is interesting to consider the special case when $C_B = C_{B+P}$,$I_B = I_{B+P}$ and $\sigma_B = \sigma_{B+P}$. In this case there are no synergies in the traditional sense. It is not surprising that when the agents are risk neutral all allocations in the

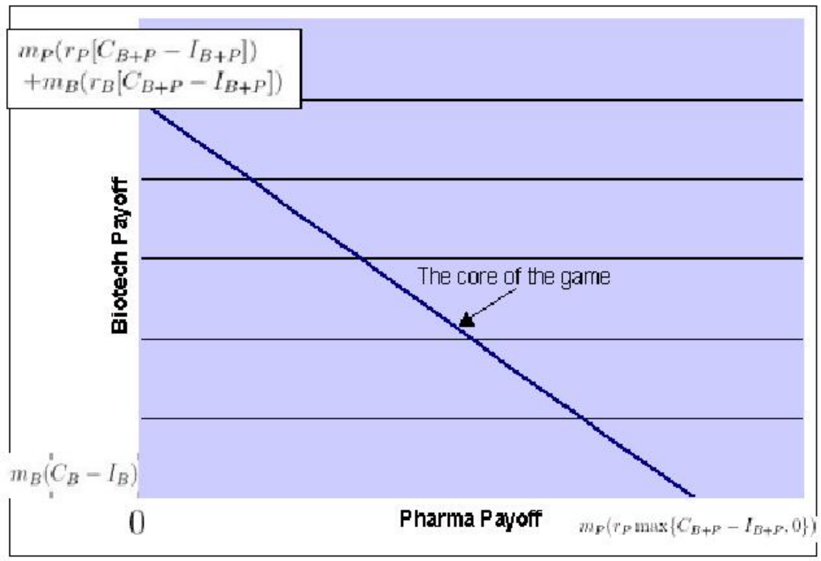


FIGURE 6. The core of the cooperative game with efficient risk sharing

biotech's core of the cooperative game have the go-alone value $C_B - I_B$, as can be seen from equation (2). Nothing is gained by a partnership.

However, the agents' risk aversion makes cooperation valuable. It can be seen from equation (7) that the core has non-empty interior. In other words, there are gains to be made by cooperating, because of risk sharing. The set of allocations of these gains amongst the players that makes no player worse off than going alone forms the new negotiation set, the 'Risk sharing' core of the deal:

$$m_B(C_B - I_B) - m_B(r_B[C_B - I_B]) \leq d_B \leq m_P(r_P[C_B - I_B]).$$

4.3. **Cooperative Options.** If the agents can wait with the launch of the project until uncertainty is resolved, we will need to solve the new cooperative game. The core will need to satisfy the following equations:

$$\begin{aligned}
m_B(r_B \max[C_{B+P} - I_{B+P}, 0]) + d_B &\geq m_B(\max(C_B - I_B, 0)) \\
m_P(r_P \max[C_{B+P} - I_{B+P}, 0]) + d_P &\geq 0 \\
d_B + d_P &= 0 \\
r_i &= \frac{\beta_i}{\sum_j \beta_j}
\end{aligned}$$

For our example of negative exponential utility functions, the risk sharing rule only depends on the risk tolerance β_i of each player and not the gamble itself. Therefore we don't need to solve the optimal risk sharing problem again, it's the same as before. For other forms of HARA utility functions, (logarithmic or power law) the optimal share of risk for each agent depends on the payoff at each state and therefore would be deferent in the presence of flexibility.
The new core of the cooperative option game can be expressed in terms of the fixed amounts that will change hands:

$$\begin{aligned}
&m_B(\max(C_B - I_B, 0)) - m_B(r_B \max[C_{B+P} - I_{B+P}, 0]) \\
&\qquad \leq d_B \leq m_P(r_P \max[C_{B+P} - I_{B+P}, 0]).
\end{aligned}$$

4.3.1. *The 'Asset' and 'Option' value of the deal.* Similarly to the risk neutral case, the asset value (static value) of the deal is the core of the deal in the absence of flexibility. Each one of these values has a risk sharing component in the sense that if the partnership has no synergies other than risk sharing, both the asset and the option cores are non-empty.

Asset core:

$$\begin{aligned}
&m_B(C_B - I_B) - m_B(r_B[C_{B+P} - I_{B+P}]) \\
&\qquad \leq d_B \leq m_P(r_P[C_{B+P} - I_{B+P}]).
\end{aligned}$$

Option core:

$$\begin{aligned}
&m_B(\max\{C_B - I_B, 0\}) - m_B(C_B - I_B) \\
&-m_B(r_B \max\{C_{B+P} - I_{B+P}, 0\} - m_B(r_B[C_{B+P} - I_{B+P}])) \leq d_B \leq \\
&\qquad m_P(r_P \max\{C_{B+P} - I_{B+P}, 0\}) - m_P(r_P[C_{B+P} - I_{B+P}]).
\end{aligned}$$

4.4. **Non-cooperative Option.** We now focus on a unilateral flexibility: The biotech has the right to opt out of codevelopment as in Section 2.3. Here we examine a more general opt-out agreement where the biotech can opt-out for a fixed amount Z (milestone payment) and royalties $r_R C_{B+P}$ on the revenue C_{B+P}. This is slightly more general than the example in Section 2.3, where we had assumed a royalty rate $r_R = 0$. The biotech company can exercise this option after uncertainty is resolved and, assuming a linear risk-sharing agrement as before, would do so if and only if $r_B(C_{B+P} - I_{B+P}) + d_B \leq Z + r_R C_{B+P}$ where r_B is the biotech's share of the revenues. The problem of finding an optimal risk share now becomes:

$$\begin{array}{ll} \max\limits_{r_B, r_P} & m_B(\max\{r_B(C_{P+B} - I_{B+P}) + d_B, Z + r_R C_{P+B}\}) \\ & + m_P(\min\{r_P(C_{P+B} - I_{B+P}) - d_B, (1 - r_R)C_{P+B} - Z\}) \\ \text{s.t.} & r_B + r_P = 1. \end{array}$$

This is a non-smooth problem, due to the max and min terms in the objective function. Furthermore, we are no longer guaranteed to find an optimal linear risk sharing rule, especially if the biotech took a substantial amount of risk in the absence of the non-cooperative option. The optimal risk sharing rule is no longer linear as the biotech sometimes takes the risk r_B and sometimes the risk r_R. One way to avoid this complication is to fix the royalties rate $r_R = r_B$ where r_B is the optimal risk share for the biotech in the absence of the option to opt-out. In this way, we allow the biotech to opt-out unilaterally but we do not affect the efficient risk allocation. In this case, the non-cooperative option will be exercised only if the amount Z, the milestone, is higher than the amount d_B the biotech receives from the pharma if it doesn't opt out. The core now satisfies the following equations:

$$\begin{array}{rcl} m_B(r_B \max[C_{B+P} - I_{B+P}, 0]) + \max(d_B, Z) & \geq & m_B(\max(C_B - I_B, 0)) \\ m_P(r_P \max[C_{B+P} - I_{B+P}, 0]) + \min(d_P, -Z) & \geq & 0 \\ d_P + d_B & = & 0 \\ r_i & = & \dfrac{\beta_i}{\beta_B + \beta_P}, \ i \in \{B, P\}. \end{array}$$

Solving for d_B will give us the core of the new game:

$$\begin{array}{c} \max\{Z, m_B(\max\{C_B - I_B, 0\}) - m_B(r_B \max\{C_{B+P} - I_{B+P}, 0\})\} \\ \leq d_B \leq m_P(r_P \max\{C_{B+P} - I_{B+P}, 0\}). \end{array}$$

The effect of such an option is illustrated in figure 7. Similarly to Section 2.3, we can distinguish three cases depending on the value of Z and the associated exercise decisions.

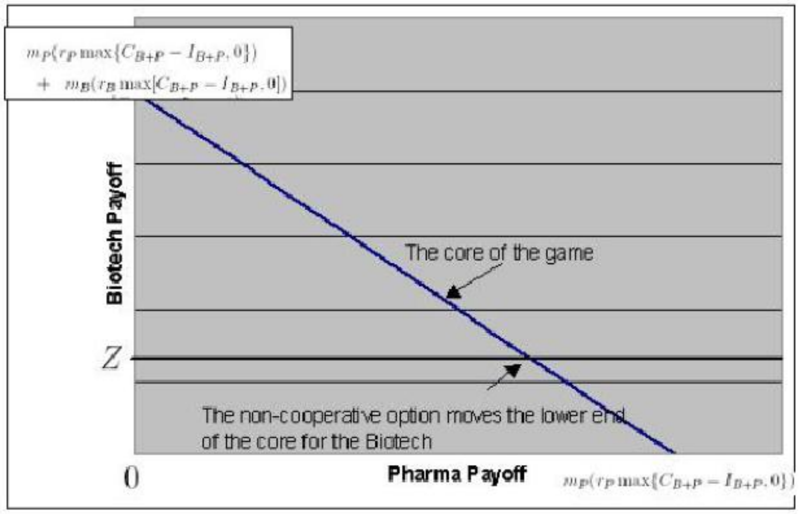
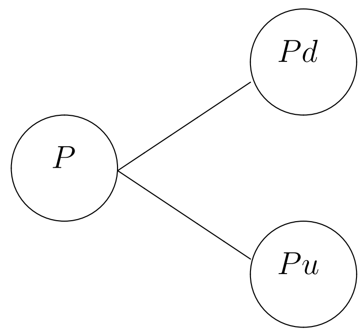


FIGURE 7. The core with non-cooperative options and efficient risk sharing

5. Complete Markets

In this final section, we will briefly illustrate the valuation of cooperative deals in the presence of complete markets. Complete markets imply that there exists a traded asset or a set of traded assets that completely span the uncertain payoffs. Trading in these assets allows the partnership to hedge all risks, making the risk preferences of each agent irrelevant. In other words, there exists a replicating portfolio. Shorting this replicating portfolio completely offsets the payoffs from the project in each state of the uncertainty. Therefore the only valuation for the investment opportunity that is consistent with the absence of arbitrage opportunities is the present value of the replicating portfolio. This is the standard method of pricing financial options, see e.g Black and Scholes [1], Hull[7].

Going back to the Biotech-Pharma example of section 2.1, we assume that the competitor in the R&D race is a one-project company whose shares are traded in the market. Since the company only has one project, the value of its stock will closely track the success or failure of their R&D project. Let's assume that in the case of success the stock price will increase from P to P_u, while the value for the biotech in this scenario will be $d_B C_B$. If the project fails the competitor's stock price will decrease to P_d, while the value for the biotech will become $u_B C_B$. This is shown in the lattice below[2].



Since the Biotech can trade in the stock of the competitor and in a risk free asset (with return r=1 for simplicity) it can create a replicating portfolio such that:

$$\begin{aligned}\psi_B P_d + \theta_B &= u_b C_B - I_B \\ \psi_B P_u + \theta_B &= d_b C_B - I_B\end{aligned}$$

where ψ is the number of shares bought and θ is the amount invested in the risk free asset. Solving for ψ and θ gives:

$$\begin{aligned}\psi_B &= C_B \frac{u_B - d_B}{P_d - P_u} \\ \theta_B &= C_B \frac{P_d d_B - u_B P_u}{P_d - P_u} - I_B\end{aligned}$$

Therefore the present value of the project if the biotech goes alone is

$$\begin{aligned}\overline{V}_B &= \psi_B P + \theta_B \\ &= \tfrac{P-P_u}{P_d-P_u} u_B C_B + (1 - \tfrac{P-P_u}{P_d-P_u}) d_B C_B - I_B \\ &= q_B u_B C_B + (1 - q_B) d_B C_B - I_B\end{aligned} \tag{8}$$

[2]Note that here we are making the assumption that the decision by the Biotech to launch their drug or not does not affect the value of the competitor. Furthermore we assume that the market capitalisation of the competitor is large enough so that any stake bought by the biotech is negligible. Although we recognise that these assumptions might be unrealistic we make them for simplicity. For models with market power see Grenadier [6], Roques and Savva[15].

This value $\overline{V}_B$ is the no arbitrage value of the project. Note that it doesn't depend on the subjective probability p as it did in section 2. The value is also independent of the risk aversion level of the agent as it is a no arbitrage price. Similarly, in the partnership case we can find a similar expression to equation (8) for the no-arbitrage value $\overline{V}_{B+P}$ of the partnership deal.

Now let us turn our attention to the negotiation set of the partnership deal. Since both agents would have to agree that the value of the project if the biotech goes alone is $\overline{V}_B$ and that the value of the partnership project is $\overline{V}_{B+P}$, the game is reduced to a deterministic cooperative game as in Section 2.1.2. The condition for the payoff to the biotech x_B to be in the core is simply

$$\overline{V}_B \leq x_B \leq \overline{V}_{B+P}.$$

The treatment of the investment opportunity with flexibility, i.e. when the partners can wait until uncertainty is resolved before committing to invest is very similar: we first price the biotech's option to invest, which is very similar to the pricing of a financial call option. Then we price the partnership's call option. By doing so we reduce the stochastic game again to a deterministic game and the core can be defined exactly as above.

6. Conclusions and further research

Our aim in this paper was to illustrate some of the effects that optionality has on the synergies created by a partnership. Needless to say, this paper leaves many questions open. There are at least three strands of interesting follow-up work: It would be useful to develop a case study, using the developed simple model or alternative real options approaches, to make the ideas relevant for practical deal negotiations. We have had some experience with the developed framework within a co-development deal negotiation between Cambridge Antibody Technology Plc., a UK-based biotech company, and Astra Zeneca and will report on this experience in another paper. More work is needed, however, to make this framework useful in practice. On the theory side, a development of the model in continuous time and for more than two players should be possible. This would provide a general framework for the investigation of options effects on partnership deal values. Finally, it would be interesting to investigate the effect of a non-cooperative Nash game following the partnership. This should shed some light on the effect of transience on partnership deals.

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